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ON THE G_2 MANIFESTATION FOR LONGITUDINALLY POLARIZED PARTICLES[1]

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Abstract

The contribution of the G_2 structure function to polarized deep inelastic scattering is slightly redefined in order to avoid kinematical zeros. Its strong Q^2-dependence implied by the Burkhardt-Cottingham (BC) sum rule naturally explains the sign change of the generalized Gerasimov-Drell-Hearn (GDH) sum rule. The status of the BC sum rule and implications for other spin processes are discussed.

PACS Numbers : 11.50.Li, 13.60.Fz, 13.60.Hb, 13.88.+e
Key-Words : sum rules, polarization, deep inelastic scattering.

February 1994
CPT-94/P.3014

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[1] This research was partly supported by INTAS (International Association for the Promotion of Cooperation with Scientists from the Independent States of the Former Soviet Union) under Contract nb 93-1180.

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1 Introduction

The structure functions G_1 and G_2 describing the spin-dependent part of deep-inelastic scattering were discussed by Feynman in his celebrated lectures [1] and the definition he used is very appealing due to the simple partonic interpretation of the dimensionless function $g_1(x)$, the only one which survives in the scaling limit for the longitudinal polarization case. However, when one studies the relatively low-Q^2 region, the alternative definition of structure functions, proposed by Schwinger a few years later [2] appears to be more useful. In fact, it was recently applied to explain the strong Q^2-dependence of the generalized Gerasimov-Drell-Hearn (GDH) sum rule[3]. The crossing point where this integral turns to zero is, to a large extent, determined by the BC sum rule. This approach was criticized in the recent paper [4], devoted to the interpolation between high and low Q^2 and a comment [5] on ref.[3] has put in focus interesting problems related to the resonance contributions.

In this article we present a systematic analysis of the BC sum rule and the G_2 manifestation in scattering with longitudinally polarized particles. The basic definitions are introduced in Section 2 and we compare them. The GDH problem is analyzed in Section 3 and the implications to the BC sum rule validity are presented in Section 4. Section 5 is devoted to the elastic and resonance contributions and to the discussion of the papers [4, 5]. In Section 6 the possible manifestations of these effects in other spin-dependent processes are discussed, while our conclusions are presented in Section 7.

2 Definitions of the spin-dependent structure functions

To define the spin-dependent structure functions one should express the antisymmetric part of the hadronic tensor $W^{\mu\nu}$ as a linear combination of all possible Lorentz-covariant tensors. These tensors should be orthogonal to the virtual photon momentum q, as required by the gauge invariance, and they are linear in the nucleon covariant polarization

s from a general property of the density matrix. If the nucleon has momentum p, we have, as usual, $sp = 0$ and $s^2 = -1$. There are only two such tensors; the first one arises already in the Born diagram

$$T_1^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} s_\alpha q_\beta \tag{1}$$

and the second tensor is just

$$T_2^{\mu\nu} = (sq)\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta. \tag{2}$$

The scalar coefficients of these tensors are specified in a well-known way

$$\begin{aligned} W_A^{\mu\nu} = \frac{-i\epsilon^{\mu\nu\alpha\beta}}{pq} q_\beta (g_1(x,Q^2)s_\alpha + g_2(x,Q^2)(s_\alpha - p_\alpha \frac{sq}{pq})) = \\ \frac{-i\epsilon^{\mu\nu\alpha\beta}}{pq} q_\beta ((g_1(x,Q^2) + g_2(x,Q^2))s_\alpha - g_2(x,Q^2)p_\alpha \frac{sq}{pq}). \end{aligned} \tag{3}$$

Here we used the dimensionless functions g_1 and g_2 whose Q^2-dependence, in the scaling region, reduces to rather weak scaling violations. However, they can also be used for low Q^2 : the Q^2-dependence is then strong and non-calculable in perturbative QCD. One can easily recover the conventional $G_1 = g_1 M^2/pq$ and $G_2 = g_2 M^4/(pq)^2$, but $g_{1,2}$ appear to be most convenient for our purposes.

In fact, considering the case of *longitudinal polarization*, one can easily transform (3) as follows:

$$W_A^{\mu\nu} = \frac{i}{M}\epsilon_\perp^{\mu\nu}(g_1(x,Q^2) - g_2(x,Q^2)\frac{Q^2M^2}{(pq)^2}), \tag{3'}$$

where $\epsilon_\perp$ is a two–dimensional antisymmetric tensor in the hyperplane orthogonal to p and q. The g_2 contribution drops either if $Q^2 = 0$, or in the scaling limit. In the later case the suppression is not exact, and leads to a higher-twist term $4g_2x^2M^2/Q^2$. Note that the coefficient of g_2 is of order unity in the resonance region and therefore the manifestation of g_2 for the elastic contribution [4] is by no means surprising.

The cancellation of g_2 is a direct consequence of the definition assumed in (3). It allows to describe the longitudinal polarization, which is kinematically dominant (the common factor $(pq)^{-1}$ in (3) is absent in (3′)) , by the single structure function g_1. However, it

confirms the fact that both tensors (1) and (2) contribute and this fact becomes important, if one is interested in the dynamical properties of g_1 and g_2.

As it was mentioned above, the T_1 tensor is "simpler": it emerges in the Born diagrams and it is the natural candidate for the application of the QCD sum rules method. Concerning the coefficient of T_2, it is strongly restricted by the Burkhardt-Cottingham sum rule and should be equal to zero "in average".

The most natural way to account for this difference is to take the coefficients of T_1 and T_2 (i.e. $g_1 + g_2$ and $-g_2$, see second line in eq.(3)) as *independent* [2]. This definition removes the mentioned kinematical zero of g_2 at $Q^2 = 0$. It seemed to us [3] that it is possible to use the same notations, just remembering that it is impossible now to cancel the g_2 terms of T_1 and T_2. However this may be misleading and it seems better to define

$$g_{1+2} = g_T \equiv g_1 + g_2. \tag{4}$$

The subscript T is the reminder of the well-known fact that only T_1 contributes to the transverse polarization case.

There is an important *physical* difference between the real and virtual photon cases. In the later case it is possible to extract from the experimental data *two* independent scalar functions (whatever they are defined): one has just to measure the asymmetries for longitudinally and transversely polarized nucleon. For real photon the transverse asymmetry is equal to zero. However, there is no reason to identify the longitudinal asymmetry with the contribution of T_1, except the kinematical zero in the standard definition. Moreover, it may be of some help to study the contributions of T_1 and T_2 separately.

3 The GDH and BC sum rules

The main problem with the generalized Gerasimov-Drell-Hearn [6, 7] sum rule is the following. Consider the Q^2-dependent integral

$$I_1(Q^2) = \frac{2M^2}{Q^2} \int_0^1 g_1(x)dx \tag{5}$$

which is defined for *all* Q^2. Note that the elastic contribution at $x = 1$ is not included. One then recovers at $Q^2 = 0$ the GDH sum rule

$$I_1(0) = -\frac{\mu_A^2}{4}, \tag{6}$$

where μ_A is the nucleon anomalous magnetic moment in nuclear magnetons. While $I_1(0)$ is always negative, its magnitude and sign at large Q^2 are determined by the Q^2 independent integral $\int_0^1 g_1(x)dx$. For the proton it is positive, so one should expect for $I_1(Q^2)$ a strong Q^2-dependence and one can ask : what is its origin?

It is possible to decompose I_1 according to the contributions of the tensors T_1 and T_2

$$I_1 = I_{1+2} - I_2, \tag{7}$$

where

$$I_{1+2}(Q^2) = \frac{2M^2}{Q^2}\int_0^1 g_{1+2}(x)dx, \qquad I_2(Q^2) = \frac{2M^2}{Q^2}\int_0^1 g_2(x)dx. \tag{8}$$

There are solid theoretical arguments to expect a strong Q^2-dependence of I_2. It is the well-known Burkhardt-Cottingham sum rule [8], derived independently by Schwinger [2] using a rather different method, i.e.,

$$I_2(Q^2) = \frac{1}{4}\mu G_M(Q^2)[\mu G_M(Q^2) - eG_E(Q^2)], \tag{9}$$

where μ is the nuclear magnetic moment, and the G's denote the familiar Sachs form factors which are dimensionless and normalised to unity at $Q^2 = 0$. For large Q^2 one can neglect the r.h.s. and one gets

$$\int_0^1 g_2(x)dx = 0. \tag{10}$$

This later equation is often called the BC sum rule. However, the elastic contribution was also present in their pioneer paper and it is this contribution which generates the strong Q^2-dependence of I_2. In particular,

$$I_2(0) = \frac{\mu_A^2 + e\mu_A}{4}, \tag{11}$$

e being the nucleon charge in elementary units, so in order to reproduce the GDH value, one should have

$$I_{1+2}(0) = \frac{e\mu_A}{4} \ . \tag{12}$$

Note that I_{1+2} does not differ from I_1 for large Q^2 due to the BC sum rule, but it is *positive* in the proton case and it is possible to obtain a smooth interpolation between large Q^2 and $Q^2 = 0$ [3].

This smooth interpolation seems to be very reasonable in the framework of the QCD sum rules method. Then one should choose some "dominant" tensor structure to study the Q^2-dependence of its scalar coefficient and T_1 appears to be a good candidate. This seems also promising from another point of view. It is not trivial to obtain within the QCD sum rules approach the GDH value at $Q^2 = 0$. Since the r.h.s. of (12) is linear in μ_A, it may be possible to obtain it using the Ward identities, just like the normalization condition for the pion form factor [9].

Concerning the neutron case, $I_{1+2}(0) = 0$ and eq.(10) naturally explain the small absolute value of g_1^n compared to g_1^p, as required by the Bjorken sum rule and confirmed by the recent SLAC and SMC measurements, despite the controversies between these two collaborations [10, 11]. The best agreement between $Q^2 = 0$ and high Q^2 is provided by the $SU(6)$ value $g_1^n = 0$.

In the proton case to give a quantitative prediction for $I_1(Q^2)$ one needs some parametrization to interpolate I_{1+2} between $Q^2 = 0$ and high Q^2. The simplest one [3] is

$$I_{1+2}(Q^2) = \theta(Q_0^2 - Q^2)(\frac{\mu_A}{4} - \frac{2M^2Q^2}{(Q_0^2)^2}\int_0^1 g_1(x)dx) + \theta(Q^2 - Q_0^2)\frac{2M^2}{Q^2}\int_0^1 g_1(x)dx. \tag{13}$$

The continuity of the function and of its derivative is guaranteed with the choice $Q_0^2 = (16M^2/\mu_A)\int_0^1 g_1(x)dx \sim 1GeV^2$, where the integral is given by the EMC data. It is quite a reasonable value to separate the perturbative and non-perturbative regions. As a result one obtains a crossing point at $Q^2 \sim 0.2GeV^2$, below the resonance region [3]. It is interesting, that the slope of I_1^p at the origin agrees with the result obtained recently in the framework of chiral perturbation theory [12].

This result is not sensitive to I_{1+2} both at low and high Q^2, provided the behavior

is smooth, but is very sensitive to the validity of BC sum rule whose possible violations have been extensively discussed in the literature [13, 14].

From another point of view, the saturation of the GDH sum rule by the contributions of low-lying resonances leads to a crossing point around $0.6 - 0.8 GeV^2$ [15]. A simultaneous assumption about the validity of the BC sum rule yields dramatic oscillations of I_{1+2} [5]. These two subtle points are the subject of the next two sections.

4 New implications for the validity of the BC sum rule

The first implication for the BC sum rule comes just from the Schwinger derivation [2]. It was obtained by using the antisymmetry property in ν, the virtual photon energy, of the relevant invariant amplitude, for which one writes a double spectral form. The antisymmetry property implies that the integration over all the kinematical region, including the elastic contribution, should give zero. This derivation is similar to the derivation of the scaling form of the BC sum rule, in the framework of the QCD twist-3 approach [16]. In this case the BC sum rule arises also as a result of the integration of an antisymmetric function over a symmetric region

$$\int_0^1 g_2(x)dx = \frac{1}{2\pi} \int_{|x,y,x-y| \le 1} dxdy \, \frac{b^A(x,y)}{x-y}. \tag{14}$$

Here $b^A(x,y)$ is the dimensionless quark-gluon correlator [17], proportional to the double spectral density in the scaling region. Its symmetry in x, y follows from T-invariance, just like ν antisymmetry in Schwinger derivation. Note, however, that the BC sum rule is not spoiled by violations of the T-invariance : $b^A(x,y)$ should be replaced by the symmetrized combination $(b^A(x,y) + b^A(y,x))/2$ (see below, Section 6).

One of the possibilities for BC sum rule violation is the *long-range* singularity $\delta(x)$ [14].If g_2 contains such a term proportional to $\delta(x)$, which can never be observed experimentally, it will give a non-zero contribution to the integral (10) and therefore will violate the BC sum rule. Note that such a situation was first noticed by Ahmed and Ross in

their pioneer paper on spin effects in QCD [18], where they also mentioned a similarity with the Schwinger term sum rule for the longitudinal structure function. However, to obtain such a behavior, one also needs to have a singularity in b^A e.g.

$$b^A(x,y) = \delta(x)\phi(x,y) + \delta(y)\phi(y,x), \tag{15}$$

where ϕ is regular function. Such a singularity should result in meaningless infinite single asymmetries, which may be generated by the correlators [17, 19].

Another implication for the BC sum rule comes from checking the sum rules (9),(12) in QED [20], as it was performed immediately after Schwinger paper. The result in QCD is the same apart from a trivial color factor.

Note that in the leading approximation μG_M is just e, while $\mu G_M - eG_E$ is the anomalous magnetic form factor. For high Q^2 it provides the elastic contribution to the BC sum rule which is decreasing like $Q^{-2} log Q^2$. However it should decrease *faster* than Q^{-2} to get the standard zero BC sum rule. Therefore, in massive on-shell one-loop QCD the BC sum rule is violated like $\alpha_s log Q^2$. Taking the leading approximation for α_s, one observes the cancellation of $log's$ and one obtains the answer

$$\int_0^1 g_2(x)dx = -\frac{C_F}{2\beta_1}, \tag{16}$$

β_1 being the one-loop beta-function. The cancellation of $log's$ is very similar to the one providing the anomalous gluon contribution to polarized DIS [21]. However, higher-order corrections and non-perturbative confinement effects should make the elastic contribution rapidly decreasing. In the standard application of QCD factorization, both elastic and inelastic contributions are included to all orders of perturbation theory. This allows to cancel the most infrared singularities and in particular the $log Q^2$ mentioned above. The result of ref.[20] means that the BC sum rule is then valid due to the cancellation of elastic and inelastic contributions. It contradicts the recent result of Mertig and van Neerven [22] who found, that the partonic BC sum rule is violated for both the $\overline{MS}$ and the on-shell renormalization schemes. Note that the later coincides with the calculations discussed above. The only difference is that the quark mass m is just a regulator of

collinear singularities, and only the terms contributing to the limit $m \to 0$ were taken into account. However, this leads to an extra divergence when $x \to 1$, regularized by an extra parameter δ, separating hard and soft gluons. As a result, a term proportional to $log\delta$ appears in the elastic contribution. However, the generalized GDH sum rule tells us that it is proportional to the anomalous magnetic moment, which is infrared stable. One may conclude, that this BC sum rule violation is an artifact of the approximation.

There is another possibility for the BC sum rule violation, namely the non-scaling one [13]. It comes from the different Regge asymptotics for the forward helicity amplitudes, related to the structure functions $G_{1,2}$. The Regge cuts from Pomeron, P' and A_2 poles could spoil the superconvergent sum rule for G_2 but this possibility is not clear at all[23].

The presence of the non-scaling BC violation does not contradict the main qualitative feature of our approach, namely, the strong Q^2-dependence of the g_2 first moment (and therefore I_2 contribution to longitudinal polarization). Although the cut contribution to the r.h.s. of BC sum rule is unknown, the same contribution should be present in I_{1+2}, because the Regge cuts do not contribute to g_1. Since it cancels out in I_1, it is still possible to say, that the qualitative origin of the rapid variation of I_1 is the elastic contribution to g_2.

The later statement leads to a quantitative prediction namely, the non-scaling violation of the BC sum rule should not affect the position of the crossing point ($0.2GeV^2$). Actually, it is natural to decompose $g_{2,1+2}$ as follows

$$g_2 = g_2^{scale} + g_2^{el} + g_2^{cut}, \quad g_{1+2} = g_{1+2}^{smooth} + g_{1+2}^{cut}. \tag{17}$$

The notations are obvious and since the first moment of g_2^{scale} is zero, the crossing point is determined by the equation

$$I_2^{el}(Q^2_{cross}) + I_2^{cut}(Q^2_{cross}) = I_{1+2}^{smooth}(Q^2_{cross}) + I_{1+2}^{cut}(Q^2_{cross}). \tag{18}$$

As it was mentioned above, the cut contributions to I_2 and I_{1+2} are equal. Moreover, to reproduce the GDH sum rule, unaffected by the cuts, I_{1+2}^{smooth} should approach the same

limit $e\mu_A/4$ at $Q^2 = 0$. One can therefore repeat the arguments of the previous section, relating the crossing point position to the elastic contribution to the BC sum rule.

The goal of future experiments is to check the BC sum rule in the low-Q^2 region and the simultaneous study of transverse and longitudinal polarizations is most appropriate for this purpose. If the cut contribution is absent, I_{1+2} directly measured for the transverse polarized target case, should change smoothly, turning to the Schwinger value $e\mu_A/4$ at $Q^2 = 0$. The cut contribution makes both I_{1+2} and I_2 change rapidly in the resonance region.

5 The resonance and elastic contributions to the GDH and BC sum rules

The only way of experimental check of the GDH sum rule yet is its saturation by the contributions of low-lying resonances. The central role here plays the $\Delta(1232)$: it provides a significant amount of GDH integral at $Q^2 = 0$ and gives a clear qualitative explanation of rapid Q^2-dependence[13]. The Δ photoproduction is dominated by the magnetic dipole form factor, leading to a negative I_1. The sign change is just related to the fast decrease of the Δ contribution.

In order to compare this picture with our approach we have separated the Δ contribution to I_{1+2} and I_2. To do this we just calculated the photoproduction Born diagram using the well-known expressions for the covariant form factors G_M, G_E and G_C. The resulting expression, obtained with the help of FORM program [25] for symbolic computations, is rather lengthy but it has a remarkable property: the leading G_M^2 term contains *only* the tensor T_2 at any Q^2. This fact is confirmed if one performs the contraction with the virtual photon density matrix: in particular, if one takes the standard definition with the kinematical zero at $Q^2 = 0$, the result should be attributed to g_1. In our approach the nonzero g_1 is due to the absence of T_1 since $g_1 = -g_2$.

This result supports qualitatively our main conclusion, namely, that the strong Q^2-

dependence of GDH sum rule should be attributed to g_2. It also naturally explains the saturation by Δ of the neutron GDH sum rule in the $SU(6)$ limit ($I_{1+2}(0) = 0$) as seen in Section 3. The quantitative difference between our result and that of the "resonance" approach may be explained in different ways. Suppose, we perform such a decomposition for the whole resonance contribution. It is a more consistent approach than to treat simultaneously the resonance approximation for I_1 and the fundamental BC sum rule for I_2. Since the covariant form factors for higher resonances are unknown, the best way may be to calculate the transverse asymmetry, using the helicity amplitudes (although one needs to know more about it than for the longitudinal one). If, nevertheless, such a decomposition is performed, three possibilities arise:

i) The results for I_2 coincide, but for I_{1+2} they are different, possibility suggested in [5]. If one really obtains this result in the above mentioned manner, it would be the first check of the BC sum rule.

ii) The results for I_{1+2} coincide but they differ for I_2; in this case it would be either an argument in favour of the violation of the BC sum rule or an indication for an additional contribution. This situation is supported by the fact, that the kink structure of I_1 at low Q^2 arises from the Δ contribution and should be therefore present in I_2 and the elastic BC contribution is monotonic.

iii) Both I_{1+2} and I_2 are different; this result would be the most unclear and perhaps the most interesting.

When discussing the resonance contributions, it seems reasonable to mention the elastic one and the main idea of ref.[4] is to interpolate between high and low Q^2 including the elastic contribution. It is quite a different problem, because one can never reach $Q^2 = 0$ and a negative GDH value: the elastic contribution is then absent for a trivial kinematical reason. Concerning the objections of this paper, all of them are based on the standard definition of g_2 with the kinematical zero.

6 Generalization to other spin-dependent processes

The main point of our analysis is the decomposition of the longitudinal polarization contribution into two pieces, one of which is directly related to g_2. It seems very easy to perform such a decomposition for an arbitrary hard process with longitudinally polarized particles.

One may also apply the QCD twist-3 approach [16], already mentioned in the previous section. The spin-dependent part of any hard cross-section may be expressed in a factorized form as

$$d\sigma_s = \int dx tr[E(x)T(x)] + \int dx_1 dx_2 tr[E_\mu(x_1, x_2)T_\mu(x_1, x_2)],$$

with

$$T(x) = M(\hat{s}\gamma^5 c_T(x)), \quad T_\mu(x_1, x_2) = \frac{M}{2\pi}(\hat{p}\gamma^5 s_\mu b_A(x_1, x_2) + i\gamma_\rho \epsilon^{\rho\mu\alpha\beta} s_\alpha p_\beta b_V(x_1, x_2)) \ . \quad (19)$$

$E^\mu(x_1, x_2)$ are density matrices of on-shell quark and quark gluon perturbative QCD diagrams. Here $c_T(x)$ and $b_{A,V}(x_1, x_2)$ are "ordinary" transverse polarized quark distributions and quark-gluon correlators, respectively. The two-argument distributions b_A and b_V are real, dimensionless and they possess symmetry properties which follow from T-invariance, i.e.

$$b_A(x_1, x_2) = b_A(x_2, x_1), \qquad b_V(x_1, x_2) = -b_V(x_2, x_1). \quad (20)$$

Note that only transverse distributions occur in the natural twist-3 basis. In the longitudinal polarization case $s^\mu = p^\mu/M$, from eq.(19) it is clear that only the correlator b_A contributes. If one makes use of the sum rule derived in [16] in terms of the longitudinal quark distribution $c_L(x)$

$$\int dx (c_L(x) - c_T(x))\sigma(x) = \frac{1}{2\pi} \int dx_1 dx_2 b_A(x_1, x_2) \frac{\sigma(x_1) - \sigma(x_2)}{x_1 - x_2}, \quad (21)$$

where $\sigma(x)$ is an arbitrary test function, one obtains the simple expression

$$d\sigma_{s,L} = \int dx tr[\hat{p}\gamma^5 E(x) c_L(x)] \ . \tag{22}$$

The sum rule (20), whose natural consequence when $\sigma(x) \equiv 1$ is the BC sum rule, provides the decomposition of the longitudinal spin-dependent quark distribution

$$c_L(x) = c_T(x) + \frac{1}{2\pi} \int dy \frac{b_A(x,y)}{x-y} \ . \tag{23}$$

While the first piece ($c_T(x)$) is proportional to $g_1 + g_2$ and we have,

$$g_1(x_B) + g_2(x_B) = c_T(x_B) + c_T(-x_B), \tag{24}$$

the second one is related to g_2 (see eq.(14)).

Although these expressions are derived for hard processes, one may also consider them as a *definition* of the parton distributions in the soft region. As the flavour summation with the target-depending weights is assumed, it seems interesting to study the GDH problem for each flavour separately.

In the absence of accurate experimental information for polarized deep inelastic scattering, one may use the indirect one, provided by the Bjorken sum rule for the difference of the proton and neutron structure functions [13]. For high Q^2, in our notations, it takes the form

$$I_1^{u-d}(Q^2) = \frac{1}{3} I_1^{p-n}(Q^2) = \frac{M^2}{3Q^2} g_A. \tag{25}$$

Here $g_A \sim 1.25$ is the axial β-decay coupling and the limit at $Q^2 = 0$ is just

$$I_1^{p-n}(0) = \frac{\mu_n^2 - \mu_p^2}{4} \ . \tag{26}$$

The important qualitative feature of these equations is the fact that I_1^{p-n} has the same (positive) sign at high Q^2 and $Q^2 = 0$, so it allows to interpolate smoothly between the two regions [13]. The transition value Q_0^2 (13) is, however, an order of magnitude larger in this case. It is interesting to study the second nonsinglet $SU(3)$ combination [26]. One

should just change $u \to d, d \to s, p \to \Xi_0, n \to \Xi_-$. Both Bjorken and GDH sum rules change sign preserving the possibility for a smooth interpolation. The transition Q^2 is lower in this case: it is of the same order $1GeV/c^2$, as for I_{1+2} in the proton case.

The main qualitative consequence of the smooth behaviour of non-singlet combinations is the following: the sharp Q^2-dependence is likely to be attributed to the $SU(3)$-singlet channel. One may expect that g_2, which is responsible for the strong Q^2-dependence, also occurs mainly in the singlet channel. This is the channel, where one has the EMC Spin Crisis and the gluon anomaly [21]. We performed the decomposition into g_{1+2} and g_2 for the box diagram, giving rise to the anomalous gluon contribution. It appears that the first moment, related to the axial anomaly, comes only from g_{1+2}, while the contribution to g_2 is zero : the BC sum rule is also respected for gluons. For higher moments the situation is less trivial : the g_{1+2} term does not contain the logarithmic corrections, and therefore may be related to the anomaly. Its x-dependence is very simple

$$E^g_{1+2} = \frac{\alpha_s}{\pi}(x-1) \ . \tag{27}$$

The *perturbative* Q^2-dependence of g_1 comes again from g_2! However these relations between the anomaly, g_2, Δ, etc. require further investigations.

The sharp dependence of the singlet combination of quark densities results in such a behaviour for each single density. One may ask, if it can be found in processes different from deep inelastic scattering. The answer, in principle, is negative. Although formula (23) is still valid, the new power corrections could appear which are absent in the DIS case. If, however, these corrections are small in comparison with the elastic BC contribution, one should expect, e.g., a decreasing of the Drell-Yan longitudinal asymmetry at $Q^2 \sim 0.2GeV^2$.

7 Conclusions

The decomposition of the longitudinally polarized particle density matrix into two pieces seems to be very simple and natural. If one starts from the very beginning with longitu-

dinal polarization, the particle is described by the single vector p^μ, leading to the single form factor $g_1\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta$ for deep inelastic scattering and to the single parton density c_L for an arbitrary hard process. Starting instead with the general case, described by the two vectors p^μ and s^μ, and going to the longitudinal polarization case $s^\mu \to p^\mu/M$, one finds oneself in a more complicated situation. It appears, that two pieces arise: the first one is related to the *transverse* polarization (g_{1+2} or c_T) and the second to the *difference* between longitudinal and transverse polarizations (g_2 or b_A).

This decomposition does not result in any physical effects, if only the high-Q^2 region is considered. However when one goes to the low-Q^2 region, there is a natural source for the g_2 strong Q^2-dependence: namely, the elastic contribution to the BC sum rule. This should not be mixed with the elastic contribution to the dispersion integral itself [4]. In this later case, the original problem with the generalized GDH sum rule does not occur anymore.

Although there is no dynamical information about g_{1+2} at low Q^2, it is *possible* to suppose its smooth behaviour and it leads to a zero for the GDH integral at $Q^2 \sim 0.2 GeV^2$. This value is just determined by the elastic BC contribution and is not affected by a possible BC violation induced by Regge cuts. The saturation of the GDH sum rule by low-lying resonances leads to substantially different values $Q^2 \sim 0.5 - 0.8 GeV^2$. It is then natural, that the simultaneous use of the BC sum rule results in a sharp and even oscillating behaviour of I_{1+2}. Moreover, the $\Delta(1232)$ contribution to g_1 via g_2 supports qualitatively a sharp Q^2-dependence of I_2. Further analysis of the resonance contributions is strongly required and in particular, it is very important to study systematically their decomposition into g_{1+2} and g_2.

The flavour dependence of the GDH sum rule suggests, that its sharp dependence is associated with the $SU(3)$ singlet channel. Therefore a possible relation to the EMC Spin Crisis via the gluonic anomaly seems to be of great interest.

Acknowledgments

One of us (O.T.) is indebted to A.P.Bakulev,A.V.Efremov and R.Ruskov for useful discussions, to B.L.Ioffe for critical remarks and to R.Workman for helpful correspondence.

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